Supplementary Material for: Regression with Missing Data, A Comparison Study of Techniques Based on Random Forests

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1 Evolution of the Missing Rate for the Other Data-Missing Mechanism

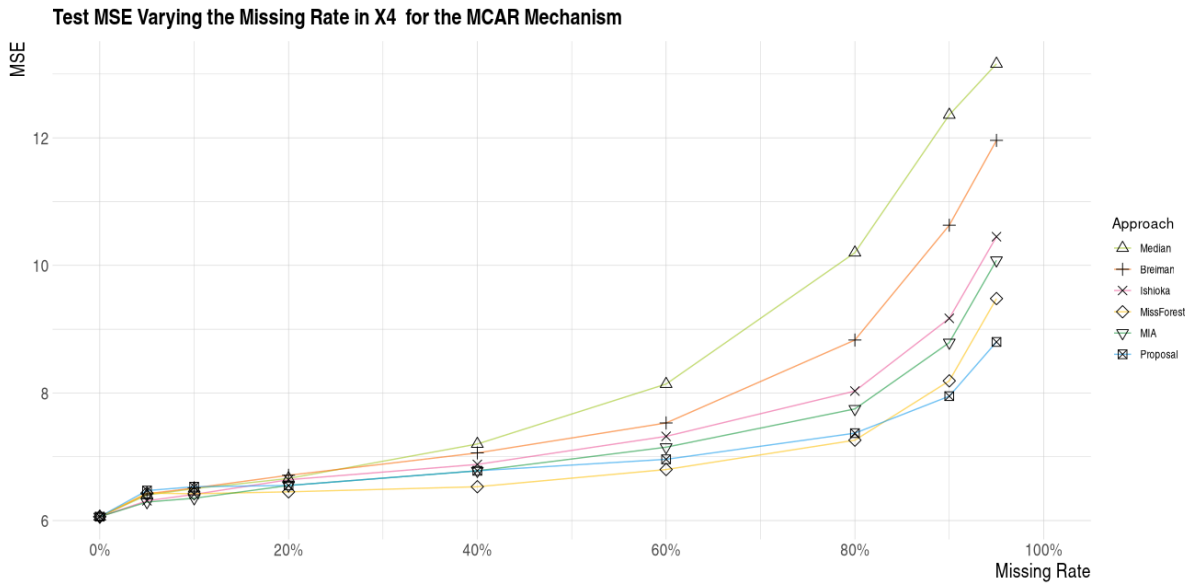

Figure 1: Average MSE for the testing data set for each percentage of missingness, considering the MCAR mechanism.

	0%	5%	10%	20%	40%
Median	6.06 ± 0.06	6.40 ± 0.06	6.50 ± 0.06	6.66 ± 0.06	7.20 ± 0.06
Breiman	6.06 ± 0.06	6.42 ± 0.06	6.51 ± 0.06	6.71 ± 0.06	7.06 ± 0.07
Ishioka	6.06 ± 0.06	6.31 ± 0.06	6.41 ± 0.06	6.64 ± 0.06	6.88 ± 0.06
MissForest	6.06 ± 0.06	6.43 ± 0.06	6.42 ± 0.05	6.45 ± 0.06	6.53 ± 0.05
MIA	6.06 ± 0.06	6.29 ± 0.06	6.35 ± 0.06	6.55 ± 0.06	6.78 ± 0.06
Proposal	6.06 ± 0.06	6.47 ± 0.05	6.53 ± 0.06	6.55 ± 0.06	6.78 ± 0.06

Table 1: Average mean squared error and its standard error for the different methods, considering the MCAR case.

	60%	80%	90%	95%
Median	8.14 ± 0.09	10.20 ± 0.14	12.36 ± 0.22	13.16 ± 0.22
Breiman	7.53 ± 0.08	8.83 ± 0.13	10.63 ± 0.23	11.96 ± 0.28
Ishioka	7.32 ± 0.07	8.03 ± 0.09	9.17 ± 0.14	10.45 ± 0.19
MissForest	6.80 ± 0.07	7.26 ± 0.07	8.19 ± 0.13	9.48 ± 0.32
MIA	7.15 ± 0.08	7.75 ± 0.09	8.79 ± 0.14	10.08 ± 0.20
Proposal	6.96 ± 0.07	7.37 ± 0.06	7.95 ± 0.07	8.80 ± 0.10

Table 2: (Cont.) Average mean squared error and its standard error for the different methods, considering the MCAR case.

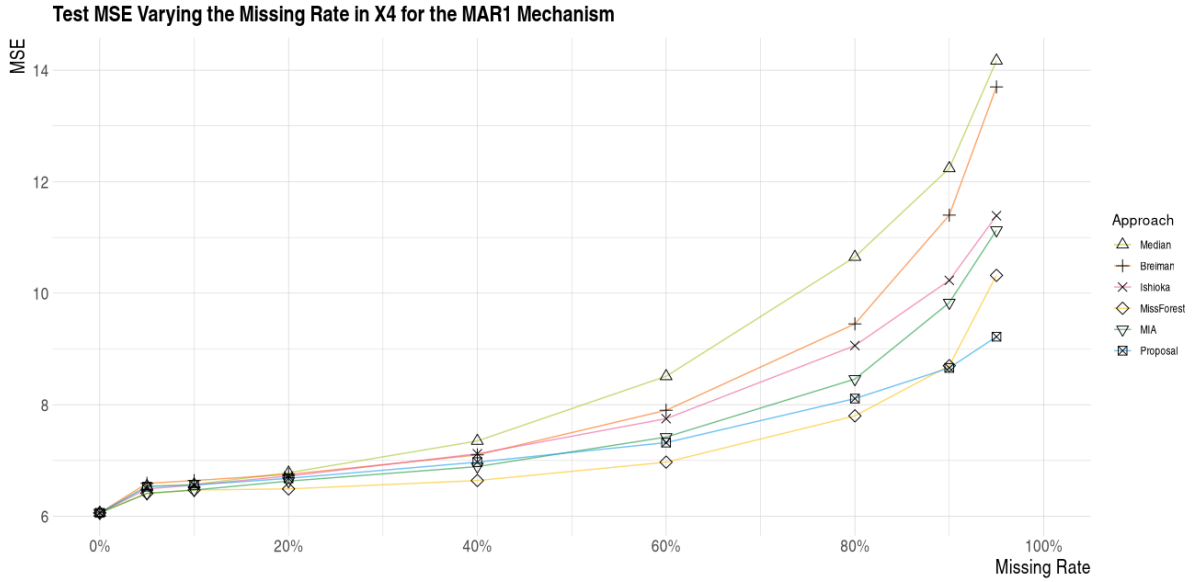

Figure 2: Average MSE for the testing data set for each percentage of missingness, considering the MAR1 mechanism.

	0%	5%	10%	20%	40%
Median	6.06 ± 0.06	6.54 ± 0.06	6.57 ± 0.05	6.78 ± 0.06	7.35 ± 0.07
Breiman	6.06 ± 0.06	6.59 ± 0.06	6.64 ± 0.06	6.75 ± 0.06	7.10 ± 0.06
Ishioka	6.06 ± 0.06	6.49 ± 0.06	6.56 ± 0.06	6.72 ± 0.06	7.12 ± 0.07
MissForest	6.06 ± 0.06	6.41 ± 0.06	6.47 ± 0.06	6.49 ± 0.06	6.64 ± 0.06
MIA	6.06 ± 0.06	6.41 ± 0.06	6.47 ± 0.06	6.63 ± 0.06	6.89 ± 0.07
Proposal	6.06 ± 0.06	6.53 ± 0.06	6.56 ± 0.06	6.68 ± 0.06	6.97 ± 0.06

Table 3: Average mean squared error and its standard error for the different methods, considering the MAR1 case.

	60%	80%	90%	95%
Median	8.51 ± 0.09	10.65 ± 0.15	12.24 ± 0.21	14.17 ± 0.28
Breiman	7.90 ± 0.10	9.45 ± 0.13	11.40 ± 0.26	13.70 ± 0.34
Ishioka	7.75 ± 0.08	9.06 ± 0.12	10.23 ± 0.13	11.39 ± 0.20
MissForest	6.97 ± 0.06	7.80 ± 0.09	8.70 ± 0.14	10.32 ± 0.35
MIA	7.42 ± 0.08	8.46 ± 0.11	9.83 ± 0.14	11.13 ± 0.20
Proposal	7.32 ± 0.07	8.11 ± 0.08	8.66 ± 0.08	9.22 ± 0.11

Table 4: (Cont.) Average mean squared error and its standard error for the different methods, considering the MAR1 case.

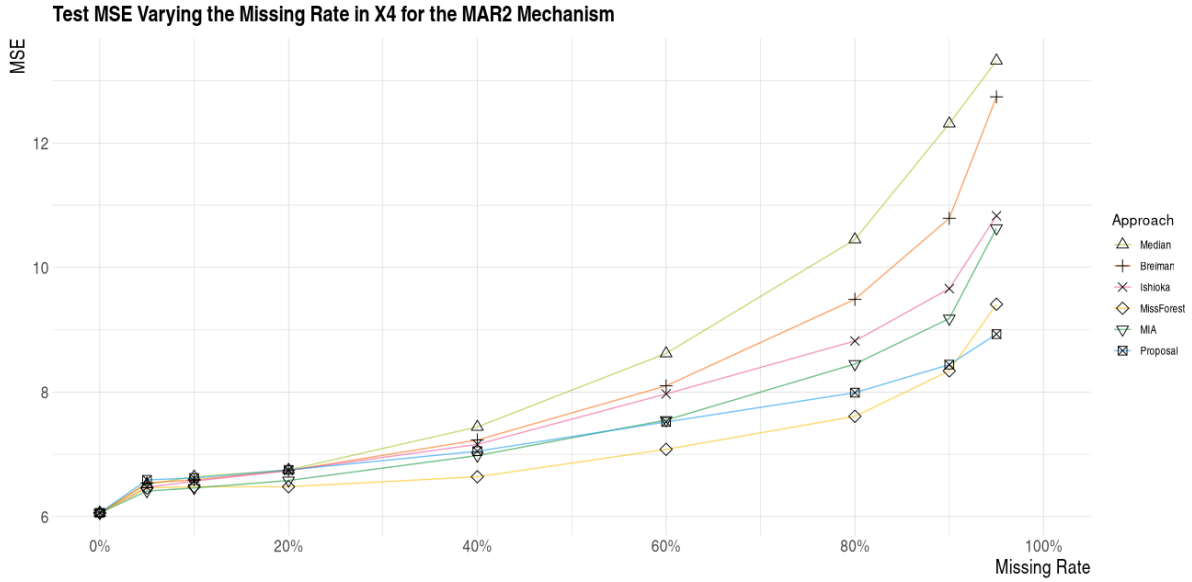

Figure 3: Average MSE for the testing data set for each percentage of missingness, considering the MAR2 mechanism.

	0%	5%	10%	20%	40%
Median	6.06 ± 0.06	6.52 ± 0.06	6.64 ± 0.06	6.75 ± 0.06	7.44 ± 0.07
Breiman	6.06 ± 0.06	6.54 ± 0.07	6.59 ± 0.06	6.74 ± 0.07	7.23 ± 0.07
Ishioka	6.06 ± 0.06	6.47 ± 0.06	6.57 ± 0.06	6.74 ± 0.07	7.16 ± 0.07
MissForest	6.06 ± 0.06	6.46 ± 0.06	6.48 ± 0.06	6.48 ± 0.06	6.64 ± 0.06
MIA	6.06 ± 0.06	6.41 ± 0.06	6.46 ± 0.06	6.58 ± 0.06	6.98 ± 0.07
Proposal	6.06 ± 0.06	6.59 ± 0.06	6.62 ± 0.06	6.75 ± 0.07	7.05 ± 0.06

Table 5: Average mean squared error and its standard error for the different methods, considering the MAR2 case.

	60%	80%	90%	95%
Median	8.62 ± 0.10	10.45 ± 0.13	12.31 ± 0.22	13.32 ± 0.23
Breiman	8.10 ± 0.09	9.49 ± 0.17	10.79 ± 0.20	12.74 ± 0.27
Ishioka	7.97 ± 0.09	8.82 ± 0.11	9.66 ± 0.14	10.83 ± 0.16
MissForest	7.08 ± 0.07	7.61 ± 0.08	8.34 ± 0.12	9.41 ± 0.25
MIA	7.55 ± 0.09	8.45 ± 0.10	9.18 ± 0.11	10.63 ± 0.18
Proposal	7.52 ± 0.07	7.99 ± 0.08	8.44 ± 0.10	8.93 ± 0.09

Table 6: (Cont.) Average mean squared error and its standard error for the different methods, considering the MAR2 case.

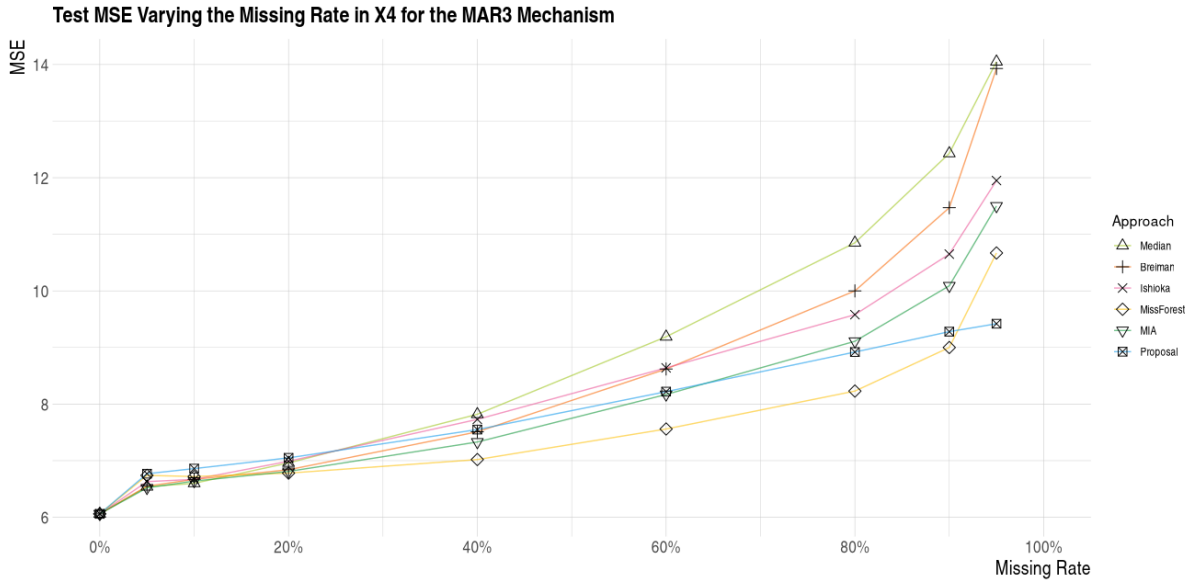

Figure 4: Average MSE for the testing data set for each percentage of missingness, considering the MAR3 mechanism.

	0	5	10	20	40
Median	6.06 ± 0.06	6.54 ± 0.06	6.60 ± 0.05	6.96 ± 0.06	7.82 ± 0.08
Breiman	6.06 ± 0.06	6.55 ± 0.06	6.67 ± 0.06	6.84 ± 0.06	7.51 ± 0.08
Ishioka	6.06 ± 0.06	6.63 ± 0.06	6.67 ± 0.06	6.99 ± 0.07	7.73 ± 0.08
MissForest	6.06 ± 0.06	6.74 ± 0.06	6.72 ± 0.06	6.78 ± 0.06	7.02 ± 0.06
MIA	6.06 ± 0.06	6.52 ± 0.06	6.54 ± 0.06	6.81 ± 0.06	7.33 ± 0.06
Proposal	6.06 ± 0.06	6.77 ± 0.06	6.86 ± 0.07	7.05 ± 0.07	7.55 ± 0.07

Table 7: Average mean squared error and its standard error for the different methods, considering the MAR3 case.

	60	80	90	95
Median	9.19 ± 0.11	10.85 ± 0.15	12.43 ± 0.20	14.05 ± 0.37
Breiman	8.62 ± 0.10	10.00 ± 0.15	11.47 ± 0.23	13.93 ± 0.27
Ishioka	8.64 ± 0.09	9.58 ± 0.10	10.65 ± 0.13	11.95 ± 0.22
MissForest	7.56 ± 0.08	8.23 ± 0.11	9.00 ± 0.15	10.67 ± 0.37
MIA	8.17 ± 0.08	9.11 ± 0.09	10.09 ± 0.12	11.50 ± 0.23
Proposal	8.22 ± 0.08	8.92 ± 0.08	9.28 ± 0.10	9.42 ± 0.10

Table 8: (Cont.) Average mean squared error and its standard error for the different methods, considering the MAR3 case.

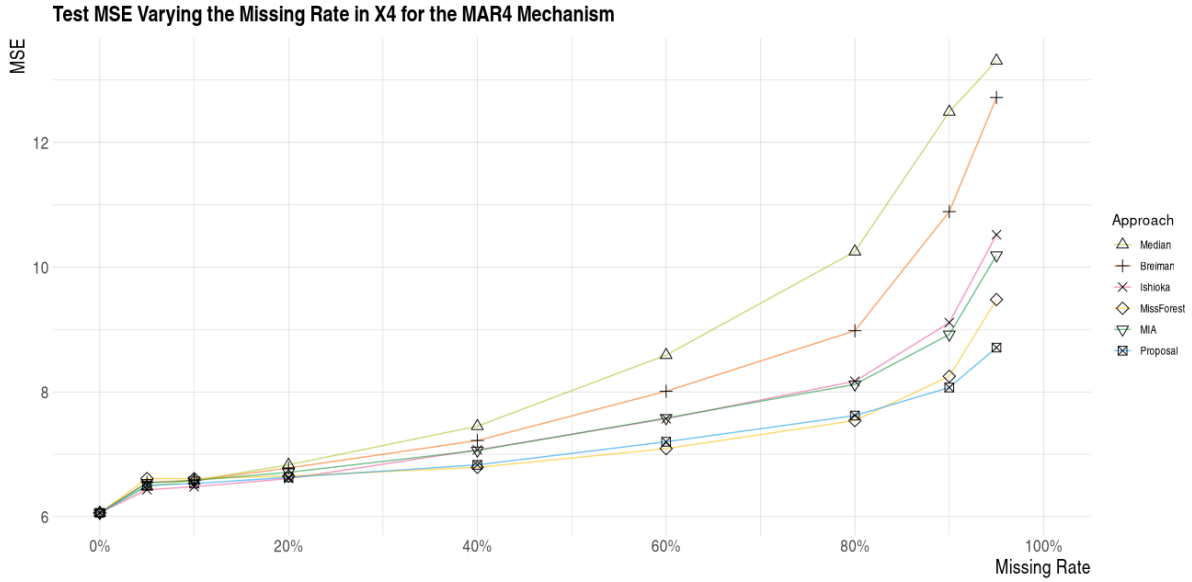

Figure 5: Average MSE for the testing data set for each percentage of missingness, considering the MAR4 mechanism.

	0	5	10	20	40
Median	6.06 ± 0.06	6.48 ± 0.06	6.58 ± 0.06	6.83 ± 0.06	7.45 ± 0.07
Breiman	6.06 ± 0.06	6.55 ± 0.06	6.58 ± 0.06	6.78 ± 0.06	7.22 ± 0.06
Ishioka	6.06 ± 0.06	6.43 ± 0.06	6.48 ± 0.06	6.61 ± 0.06	7.07 ± 0.08
MissForest	6.06 ± 0.06	6.61 ± 0.06	6.61 ± 0.06	6.65 ± 0.06	6.79 ± 0.06
MIA	6.06 ± 0.06	6.54 ± 0.06	6.58 ± 0.06	6.71 ± 0.06	7.06 ± 0.06
Proposal	6.06 ± 0.06	6.50 ± 0.06	6.53 ± 0.05	6.63 ± 0.06	6.83 ± 0.06

Table 9: Average mean squared error and its standard error for the different methods, considering the MAR4 case.

	60	80	90	95
Median	8.59 ± 0.10	10.25 ± 0.14	12.49 ± 0.23	13.31 ± 0.27
Breiman	8.01 ± 0.09	8.98 ± 0.11	10.82 ± 0.25	12.72 ± 0.37
Ishioka	7.57 ± 0.07	8.17 ± 0.08	9.11 ± 0.14	10.52 ± 0.15
MissForest	7.09 ± 0.06	7.54 ± 0.06	8.25 ± 0.12	9.48 ± 0.37
MIA	7.58 ± 0.08	8.12 ± 0.08	8.92 ± 0.13	10.19 ± 0.23
Proposal	7.20 ± 0.06	7.62 ± 0.07	8.07 ± 0.07	8.71 ± 0.10

Table 10: (Cont.) Average mean squared error and its standard error for the different methods, considering the MAR4 case.

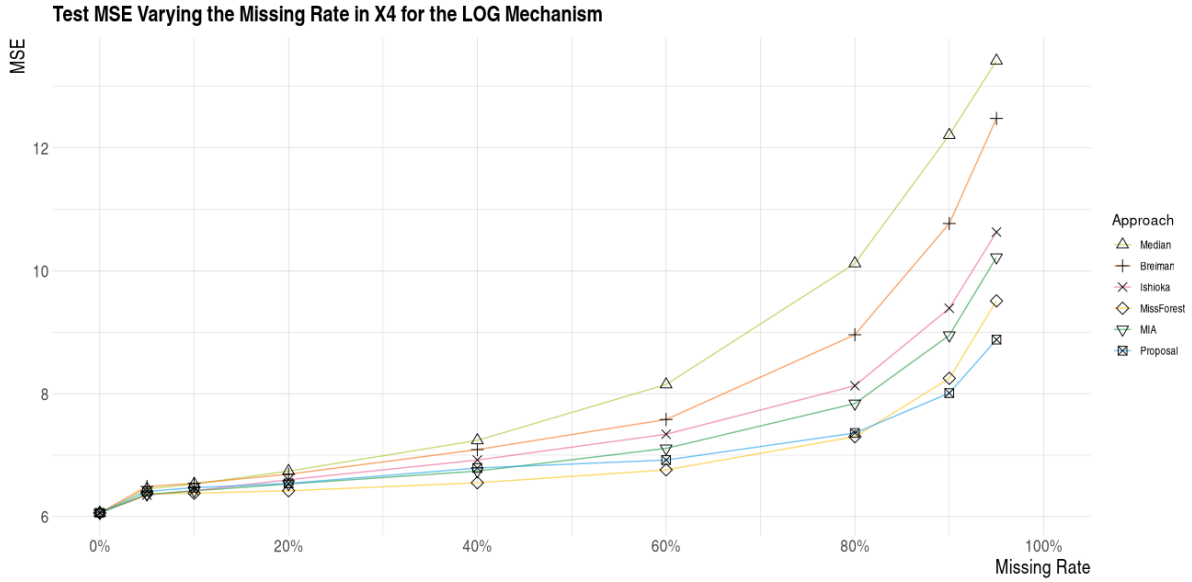

Figure 6: Average MSE for the testing data set for each percentage of missingness, considering the LOG mechanism.

	0	5	10	20	40
Median	6.06 ± 0.06	6.45 ± 0.06	6.53 ± 0.07	6.74 ± 0.07	7.24 ± 0.08
Breiman	6.06 ± 0.06	6.49 ± 0.06	6.54 ± 0.06	6.69 ± 0.07	7.09 ± 0.08
Ishioka	6.06 ± 0.06	6.35 ± 0.06	6.42 ± 0.06	6.60 ± 0.06	6.92 ± 0.07
MissForest	6.06 ± 0.06	6.37 ± 0.06	6.38 ± 0.06	6.42 ± 0.06	6.55 ± 0.06
MIA	6.06 ± 0.06	6.36 ± 0.06	6.42 ± 0.06	6.53 ± 0.06	6.74 ± 0.07
Proposal	6.06 ± 0.06	6.41 ± 0.06	6.47 ± 0.06	6.54 ± 0.06	6.79 ± 0.07

Table 11: Average mean squared error and its standard error for the different methods, considering the LOG case.

	60	80	90	95
Median	8.15 ± 0.08	10.12 ± 0.15	12.21 ± 0.22	13.42 ± 0.25
Breiman	7.58 ± 0.08	8.96 ± 0.16	10.77 ± 0.24	12.48 ± 0.31
Ishioka	7.34 ± 0.08	8.13 ± 0.10	9.39 ± 0.15	10.63 ± 0.19
MissForest	6.76 ± 0.06	7.30 ± 0.07	8.25 ± 0.11	9.51 ± 0.25
MIA	7.11 ± 0.08	7.84 ± 0.10	8.95 ± 0.15	10.22 ± 0.18
Proposal	6.92 ± 0.07	7.36 ± 0.07	8.01 ± 0.08	8.88 ± 0.09

Table 12: (Cont.) Average mean squared error and its standard error for the different methods, considering the LOG case.

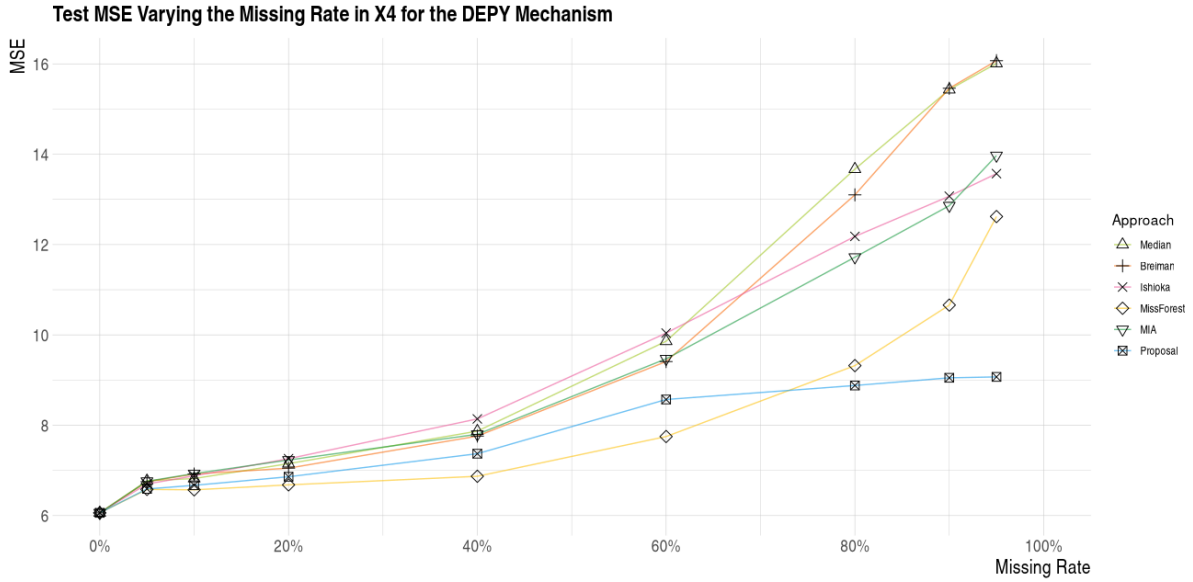

Figure 7: Average MSE for the testing data set for each percentage of missingness, considering the DEPY mechanism.

	0	5	10	20	40
Median	6.06 ± 0.06	6.77 ± 0.06	6.82 ± 0.06	7.15 ± 0.07	7.87 ± 0.08
Breiman	6.06 ± 0.06	6.75 ± 0.06	6.92 ± 0.07	7.05 ± 0.07	7.76 ± 0.08
Ishioka	6.06 ± 0.06	6.70 ± 0.07	6.88 ± 0.07	7.26 ± 0.07	8.14 ± 0.09
MissForest	6.06 ± 0.06	6.58 ± 0.06	6.57 ± 0.06	6.68 ± 0.06	6.87 ± 0.07
MIA	6.06 ± 0.06	6.76 ± 0.07	6.93 ± 0.07	7.23 ± 0.08	7.80 ± 0.08
Proposal	6.06 ± 0.06	6.59 ± 0.06	6.67 ± 0.06	6.86 ± 0.06	7.37 ± 0.07

Table 13: Average mean squared error and its standard error for the different methods, considering the DEPY case.

	60	80	90	95
Median	9.86 ± 0.13	13.67 ± 0.20	15.43 ± 0.23	16.01 ± 0.26
Breiman	9.41 ± 0.12	13.10 ± 0.20	15.46 ± 0.27	16.07 ± 0.25
Ishioka	10.04 ± 0.13	12.18 ± 0.14	13.07 ± 0.19	13.57 ± 0.22
MissForest	7.75 ± 0.08	9.32 ± 0.16	10.66 ± 0.31	12.62 ± 0.54
MIA	9.47 ± 0.12	11.72 ± 0.15	12.86 ± 0.16	13.97 ± 0.27
Proposal	8.57 ± 0.11	8.88 ± 0.10	9.05 ± 0.09	9.07 ± 0.09

Table 14: (Cont.) Average mean squared error and its standard error for the different methods, considering the DEPY case.

2 BIAS

	0	5	10	20	40
Median	0.00 ± 0.02	-0.01 ± 0.02	-0.01 ± 0.02	-0.01 ± 0.02	0.01 ± 0.03
Breiman	0.00 ± 0.02	0.01 ± 0.02	-0.03 ± 0.02	-0.04 ± 0.02	-0.05 ± 0.03
Ishioka	0.00 ± 0.02	-0.05 ± 0.02	-0.05 ± 0.02	-0.04 ± 0.02	-0.03 ± 0.02
MissForest	0.00 ± 0.02	0.00 ± 0.02	-0.02 ± 0.02	-0.01 ± 0.02	-0.01 ± 0.02
MIA	0.00 ± 0.02	0.00 ± 0.02	-0.02 ± 0.02	0.00 ± 0.02	0.00 ± 0.02
Proposal	0.00 ± 0.02	0.00 ± 0.02	0.00 ± 0.02	0.00 ± 0.02	0.01 ± 0.02

Table 15: Average bias and its standard error for the different methods, considering the MCAR case.

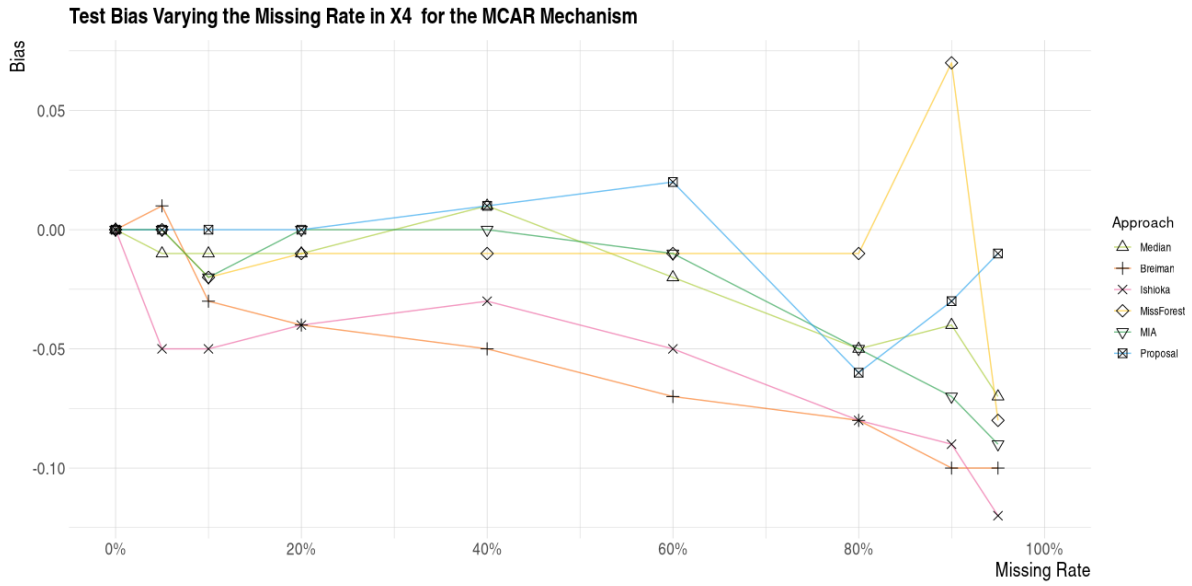

Figure 8: Average Bias for the testing data set for each percentage of missingness, considering the MCAR mechanism

	60	80	90	95
Median	-0.02 ± 0.03	-0.05 ± 0.03	-0.04 ± 0.05	-0.07 ± 0.06
Breiman	-0.07 ± 0.03	-0.08 ± 0.03	-0.10 ± 0.03	-0.10 ± 0.04
Ishioka	-0.05 ± 0.03	-0.08 ± 0.03	-0.09 ± 0.04	-0.12 ± 0.05
MissForest	-0.01 ± 0.03	-0.01 ± 0.04	0.07 ± 0.05	-0.08 ± 0.07
MIA	-0.01 ± 0.03	-0.05 ± 0.03	-0.07 ± 0.05	-0.09 ± 0.07
Proposal	0.02 ± 0.03	-0.06 ± 0.03	-0.03 ± 0.04	-0.01 ± 0.05

Table 16: (Cont.) Average bias and its standard error for the different methods, considering the MCAR case.

	0	5	10	20	40
Median	0.00 ± 0.02	-0.06 ± 0.02	-0.08 ± 0.02	-0.09 ± 0.02	-0.20 ± 0.03
Breiman	0.00 ± 0.02	-0.04 ± 0.02	-0.04 ± 0.02	-0.05 ± 0.03	-0.09 ± 0.03
Ishioka	0.00 ± 0.02	-0.04 ± 0.02	-0.05 ± 0.02	-0.07 ± 0.02	-0.16 ± 0.03
MissForest	0.00 ± 0.02	-0.05 ± 0.02	-0.06 ± 0.02	-0.07 ± 0.02	-0.13 ± 0.02
MIA	0.00 ± 0.02	-0.08 ± 0.02	-0.12 ± 0.02	-0.14 ± 0.03	-0.26 ± 0.03
Proposal	0.00 ± 0.02	-0.08 ± 0.02	-0.09 ± 0.02	-0.10 ± 0.02	-0.20 ± 0.03

Table 17: Average bias and its standard error for the different methods, considering the MAR1 case.

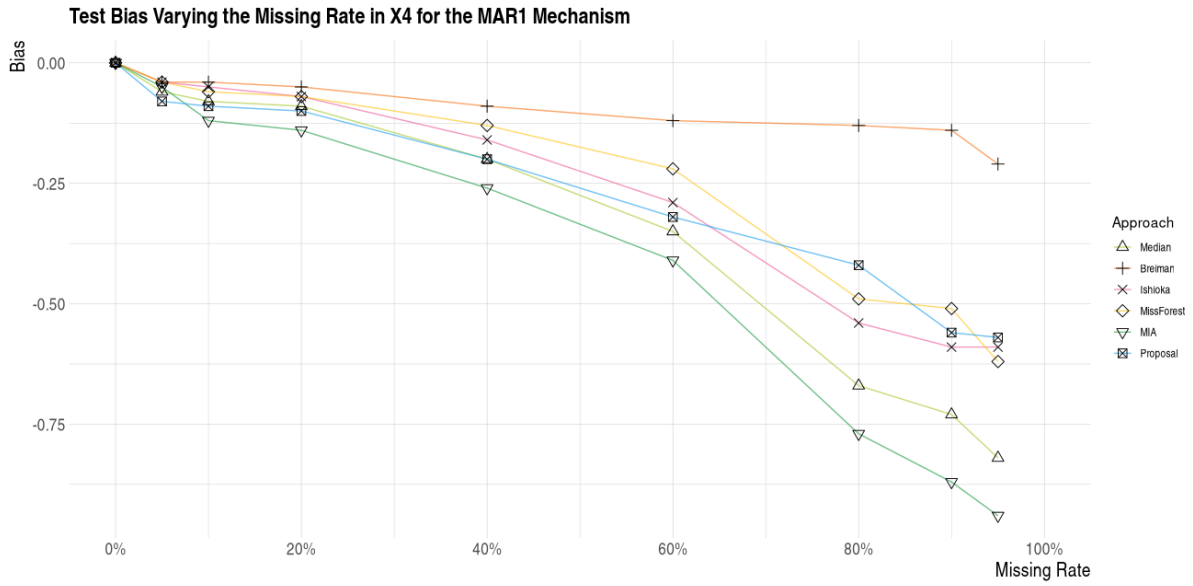

Figure 9: Average bias for the testing data set for each percentage of missingness, considering the MAR1 mechanism.

	60	80	90	95
Median	-0.35 ± 0.03	-0.67 ± 0.03	-0.73 ± 0.05	-0.82 ± 0.04
Breiman	-0.12 ± 0.03	-0.13 ± 0.03	-0.14 ± 0.03	-0.21 ± 0.05
Ishioka	-0.29 ± 0.03	-0.54 ± 0.03	-0.59 ± 0.05	-0.59 ± 0.05
MissForest	-0.22 ± 0.03	-0.49 ± 0.04	-0.51 ± 0.05	-0.62 ± 0.07
MIA	-0.41 ± 0.03	-0.77 ± 0.03	-0.87 ± 0.05	-0.94 ± 0.05
Proposal	-0.32 ± 0.03	-0.42 ± 0.04	-0.56 ± 0.03	-0.57 ± 0.03

Table 18: (Cont.) Average bias and its standard error for the different methods, considering the MAR1 case.

	0	5	10	20	40
Median	0.00 ± 0.02	-0.07 ± 0.02	-0.09 ± 0.02	-0.15 ± 0.02	-0.28 ± 0.02
Breiman	0.00 ± 0.02	-0.03 ± 0.02	-0.05 ± 0.02	-0.08 ± 0.02	-0.08 ± 0.03
Ishioka	0.00 ± 0.02	-0.04 ± 0.02	-0.06 ± 0.02	-0.10 ± 0.02	-0.22 ± 0.02
MissForest	0.00 ± 0.02	-0.05 ± 0.02	-0.06 ± 0.02	-0.10 ± 0.02	-0.16 ± 0.02
MIA	0.00 ± 0.02	-0.08 ± 0.02	-0.12 ± 0.02	-0.17 ± 0.02	-0.36 ± 0.02
Proposal	0.01 ± 0.02	-0.10 ± 0.02	-0.11 ± 0.02	-0.15 ± 0.02	-0.25 ± 0.02

Table 19: Average bias and its standard error for the different methods, considering the MAR2 case.

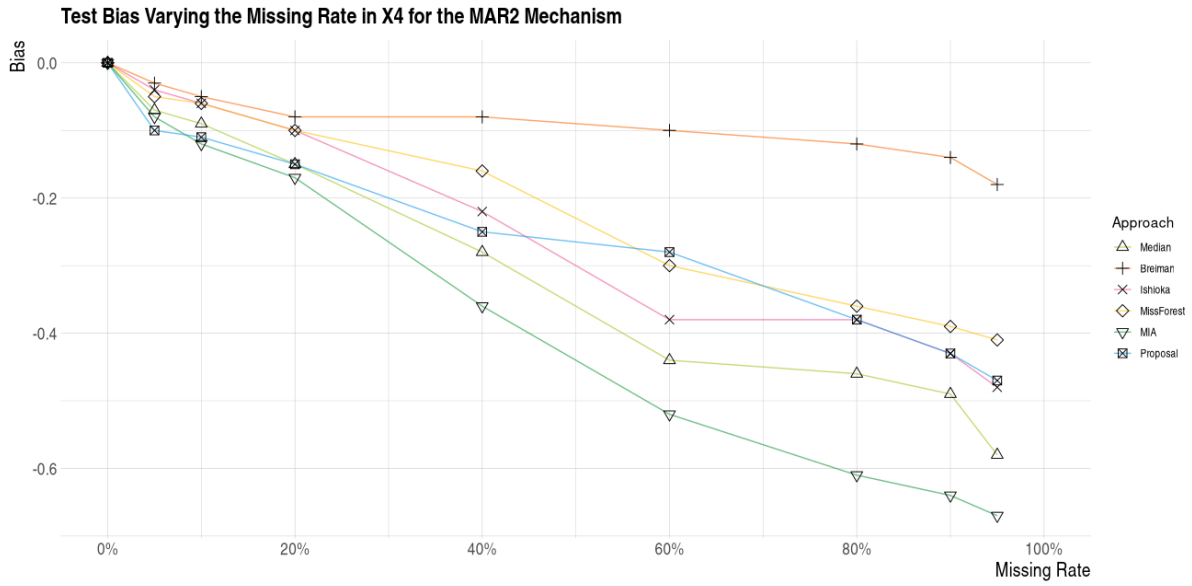

Figure 10: Average bias for the testing data set for each percentage of missingness, considering the MAR2 mechanism.

	60	80	90	95
Median	-0.44 ± 0.03	-0.46 ± 0.05	-0.49 ± 0.04	-0.58 ± 0.03
Breiman	-0.10 ± 0.03	-0.12 ± 0.03	-0.14 ± 0.03	-0.18 ± 0.03
Ishioka	-0.38 ± 0.03	-0.38 ± 0.03	-0.43 ± 0.04	-0.48 ± 0.05
MissForest	-0.30 ± 0.03	-0.36 ± 0.04	-0.39 ± 0.05	-0.41 ± 0.06
MIA	-0.52 ± 0.03	-0.61 ± 0.03	-0.64 ± 0.05	-0.67 ± 0.06
Proposal	-0.28 ± 0.04	-0.38 ± 0.02	-0.43 ± 0.03	-0.47 ± 0.04

Table 20: (Cont.) Average bias and its standard error for the different methods, considering the MAR2 case.

	0	5	10	20	40
Median	0.00 ± 0.02	-0.11 ± 0.02	-0.15 ± 0.02	-0.23 ± 0.02	-0.44 ± 0.02
Breiman	0.00 ± 0.02	-0.04 ± 0.02	-0.06 ± 0.02	-0.08 ± 0.02	-0.12 ± 0.02
Ishioka	0.00 ± 0.02	-0.05 ± 0.02	-0.11 ± 0.02	-0.19 ± 0.02	-0.35 ± 0.03
MissForest	0.00 ± 0.02	-0.09 ± 0.02	-0.13 ± 0.02	-0.17 ± 0.02	-0.29 ± 0.02
MIA	0.00 ± 0.02	-0.16 ± 0.02	-0.18 ± 0.02	-0.29 ± 0.02	-0.52 ± 0.03
Proposal	0.00 ± 0.02	-0.17 ± 0.02	-0.19 ± 0.02	-0.27 ± 0.02	-0.43 ± 0.02

Table 21: Average bias and its standard error for the different methods, considering the MAR3 case.

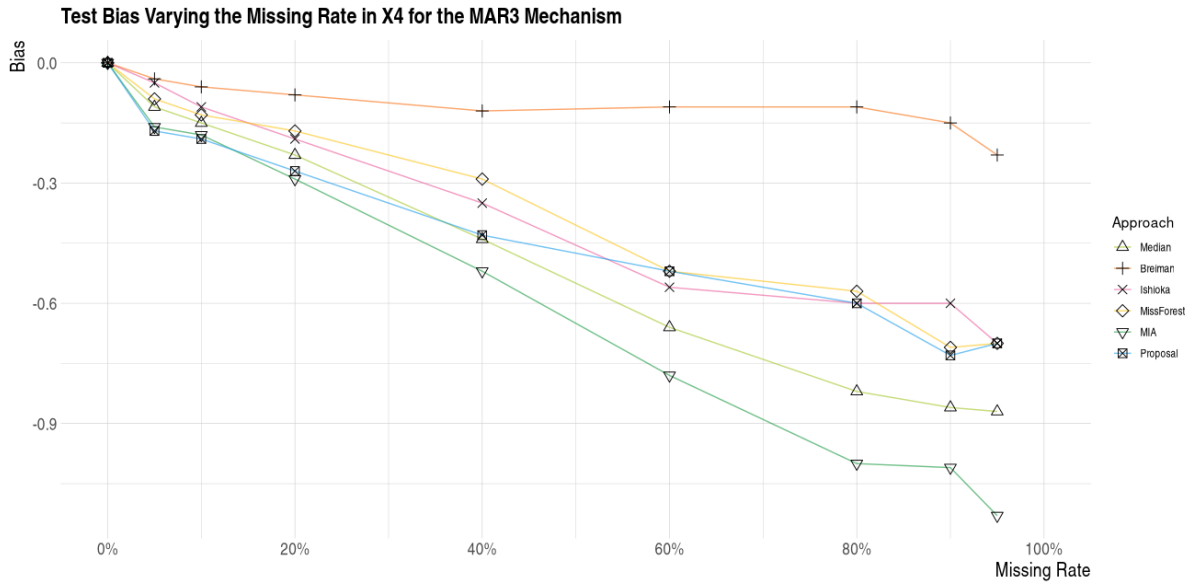

Figure 11: Average bias for the testing data set for each percentage of missingness, considering the MAR3 mechanism.

	60	80	90	95
Median	-0.66 ± 0.03	-0.82 ± 0.05	-0.86 ± 0.03	-0.87 ± 0.04
Breiman	-0.11 ± 0.03	-0.11 ± 0.03	-0.15 ± 0.03	-0.23 ± 0.04
Ishioka	-0.56 ± 0.03	-0.60 ± 0.03	-0.60 ± 0.04	-0.70 ± 0.05
MissForest	-0.52 ± 0.03	-0.57 ± 0.04	-0.71 ± 0.06	-0.70 ± 0.06
MIA	-0.78 ± 0.03	-1.00 ± 0.03	-1.01 ± 0.05	-1.13 ± 0.05
Proposal	-0.52 ± 0.04	-0.60 ± 0.03	-0.73 ± 0.03	-0.70 ± 0.04

Table 22: (Cont.) Average bias and its standard error for the different methods, considering the MAR3 case.

	0	5	10	20	40
Median	0.00 ± 0.02	0.04 ± 0.02	0.09 ± 0.02	0.15 ± 0.02	0.14 ± 0.02
Breiman	0.00 ± 0.02	0.09 ± 0.02	0.08 ± 0.02	0.08 ± 0.02	0.12 ± 0.02
Ishioka	0.00 ± 0.02	-0.01 ± 0.02	0.05 ± 0.02	0.08 ± 0.02	0.09 ± 0.02
MissForest	0.00 ± 0.02	0.05 ± 0.02	0.07 ± 0.02	0.08 ± 0.02	0.09 ± 0.02
MIA	0.00 ± 0.02	0.12 ± 0.02	0.17 ± 0.02	0.24 ± 0.02	0.26 ± 0.02
Proposal	0.00 ± 0.02	0.04 ± 0.02	0.04 ± 0.02	0.13 ± 0.02	0.13 ± 0.02

Table 23: Average bias and its standard error for the different methods, considering the MAR4 case.

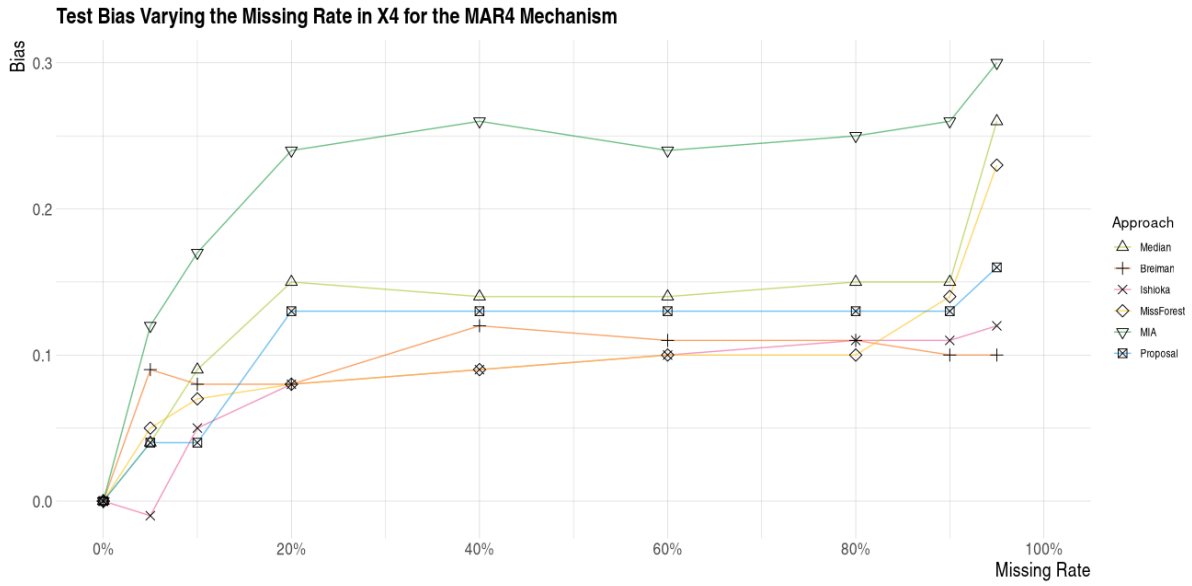

Figure 12: Average bias for the testing data set for each percentage of missingness, considering the MAR4 mechanism.

	60	80	90	95
Median	0.14 ± 0.03	0.15 ± 0.03	0.15 ± 0.04	0.26 ± 0.05
Breiman	0.11 ± 0.03	0.11 ± 0.03	0.10 ± 0.03	0.10 ± 0.04
Ishioka	0.10 ± 0.03	0.11 ± 0.03	0.11 ± 0.04	0.12 ± 0.06
MissForest	0.10 ± 0.03	0.10 ± 0.04	0.14 ± 0.05	0.23 ± 0.07
MIA	0.24 ± 0.03	0.25 ± 0.04	0.26 ± 0.05	0.30 ± 0.06
Proposal	0.13 ± 0.03	0.13 ± 0.03	0.13 ± 0.04	0.16 ± 0.03

Table 24: (Cont.) Average bias and its standard error for the different methods, considering the MAR4 case.

	0	5	10	20	40
Median	0.00 ± 0.02	0.01 ± 0.02	0.00 ± 0.02	-0.01 ± 0.02	-0.05 ± 0.02
Breiman	0.00 ± 0.02	0.01 ± 0.02	0.01 ± 0.02	-0.01 ± 0.02	-0.03 ± 0.03
Ishioka	0.00 ± 0.02	0.01 ± 0.02	0.00 ± 0.02	-0.02 ± 0.02	-0.03 ± 0.03
MissForest	0.00 ± 0.02	0.00 ± 0.02	0.00 ± 0.02	0.00 ± 0.02	-0.04 ± 0.02
MIA	0.00 ± 0.02	0.03 ± 0.02	0.02 ± 0.02	-0.01 ± 0.03	-0.07 ± 0.03
Proposal	0.01 ± 0.02	-0.01 ± 0.02	-0.01 ± 0.02	-0.03 ± 0.02	-0.08 ± 0.02

Table 25: Average bias and its standard error for the different methods, considering the LOG case.

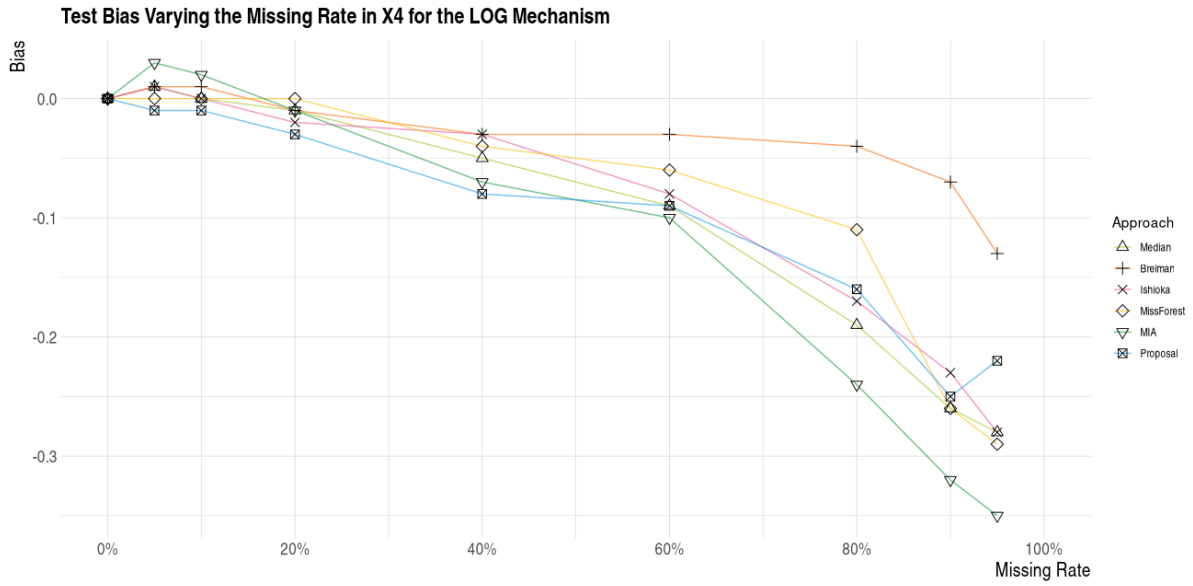

Figure 13: Average bias for the testing data set for each percentage of missingness, considering the LOG mechanism.

	60	80	90	95
Median	-0.09 ± 0.03	-0.19 ± 0.03	-0.26 ± 0.05	-0.28 ± 0.05
Breiman	-0.03 ± 0.03	-0.04 ± 0.03	-0.07 ± 0.03	-0.13 ± 0.04
Ishioka	-0.08 ± 0.03	-0.17 ± 0.03	-0.23 ± 0.05	-0.28 ± 0.05
MissForest	-0.06 ± 0.03	-0.11 ± 0.03	-0.26 ± 0.06	-0.29 ± 0.07
MIA	-0.10 ± 0.03	-0.24 ± 0.04	-0.32 ± 0.05	-0.35 ± 0.06
Proposal	-0.09 ± 0.03	-0.16 ± 0.03	-0.25 ± 0.04	-0.22 ± 0.04

Table 26: (Cont.) Average bias and its standard error for the different methods, considering the LOG case.

	0	5	10	20	40
Median	0.00 ± 0.02	0.11 ± 0.02	0.12 ± 0.02	0.19 ± 0.02	0.37 ± 0.02
Breiman	0.00 ± 0.02	0.04 ± 0.02	0.04 ± 0.02	0.07 ± 0.02	0.08 ± 0.03
Ishioka	0.00 ± 0.02	0.06 ± 0.02	0.06 ± 0.02	0.12 ± 0.02	0.23 ± 0.03
MissForest	0.00 ± 0.02	0.06 ± 0.02	0.05 ± 0.02	0.08 ± 0.02	0.11 ± 0.02
MIA	0.00 ± 0.02	0.37 ± 0.02	0.40 ± 0.02	0.54 ± 0.02	0.78 ± 0.03
Proposal	0.00 ± 0.02	0.20 ± 0.02	0.22 ± 0.02	0.29 ± 0.02	0.34 ± 0.04

Table 27: Average bias and its standard error for the different methods, considering the DEPY case.

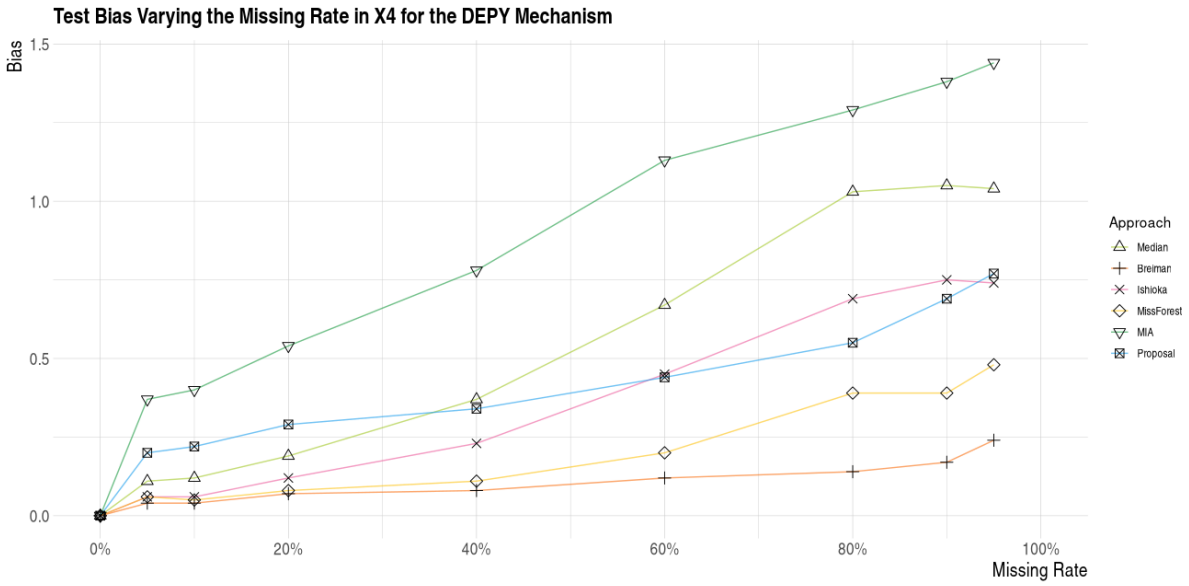

Figure 14: Average bias for the testing data set for each percentage of missingness, considering the DEPY mechanism.

	60	80	90	95
Median	0.67 ± 0.03	1.03 ± 0.03	1.05 ± 0.03	1.04 ± 0.05
Breiman	0.12 ± 0.03	0.14 ± 0.03	0.17 ± 0.03	0.24 ± 0.04
Ishioka	0.45 ± 0.03	0.69 ± 0.03	0.75 ± 0.03	0.74 ± 0.04
MissForest	0.20 ± 0.03	0.39 ± 0.03	0.39 ± 0.03	0.48 ± 0.06
MIA	1.13 ± 0.03	1.44 ± 0.03	1.38 ± 0.03	1.29 ± 0.04
Proposal	0.44 ± 0.03	0.55 ± 0.04	0.69 ± 0.03	0.77 ± 0.03

Table 28: (Cont.) Average bias and its standard error for the different methods, considering the DEPY case.